Nonlinear Functional Analysis In Banach Spaces And Banach Algebras Fixed Point Theory Under Weak Topology For Nonlinear Operators And Block Operator And Research Notes In Mathematics

Applied Nonlinear Functional Analysis

Banach spaces provide a framework for linear and nonlinear functional analysis, operator theory, abstract analysis, probability, optimization and other branches of mathematics. This book introduces the reader to linear functional analysis and to related parts of infinite-dimensional Banach space theory. Key Features: - Develops classical theory, including weak topologies, locally convex space, Schauder bases and compact operator theory - Covers Radon-Nikodým property, finite-dimensional spaces and local theory on tensor products - Contains sections on uniform homeomorphisms and non-linear theory, Rosenthal's L1 theorem, fixed points, and more - Includes information about further topics and directions of research and some open problems at the end of each chapter - Provides numerous exercises for practice The text is suitable for graduate courses or for independent study. Prerequisites include basic courses in calculus and linear. Researchers in functional analysis will also benefit for this book as it can serve as a reference book.

Fixed Points of Nonlinear Operators
This self-contained textbook provides the basic, abstract tools used in nonlinear analysis and their applications to semilinear elliptic boundary value problems and displays how various approaches can easily be applied to a range of model cases. Complete with a preliminary chapter, an appendix that includes further results on weak derivatives, and chapter-by-chapter exercises, this book is a practical text for an introductory course or seminar on nonlinear functional analysis.

**Nonlinear Functional Analysis and Its Applications**

This book introduces the basic principles of functional analysis and areas of Banach space theory that are close to nonlinear analysis and topology. The text can be used in graduate courses or for independent study. It includes a large number of exercises of different levels of difficulty, accompanied by hints.

**Banach Space Theory**

As long as a branch of knowledge offers an abundance of problems, it is full of vitality. David Hilbert Over the last 15 years I have given lectures on a variety of problems in nonlinear functional analysis and its applications. In doing this, I have recommended to my students a number of excellent monographs devoted to specialized topics, but there was no complete survey-type exposition of nonlinear functional analysis making available a quick survey to the wide range of readers including mathematicians, natural scientists, and engineers who have only an elementary knowledge of linear functional analysis. I have tried to close this gap with my five-part lecture notes, the first three parts of which have been published in the Teubner-Texte series by Teubner-Verlag, Leipzig, 1976, 1977, and 1978. The present English edition was translated from a completely rewritten manuscript which is significantly longer than the original version in the Teubner-Texte series. The material is organized in the following way: Part I: Fixed Point Theorems. Part II: Monotone Operators. Part III: Variational Methods and Optimization. Parts IV jV: Applications to Mathematical Physics. The exposition is guided by the following considerations: (a) What are the supporting basic ideas and what intrinsic interrelations exist between them? (/3) In what relation do the basic ideas stand to the known propositions of classical analysis and linear functional analysis? (y) What typical applications are there? VII Preface viii Special emphasis is placed on motivation.

**Nonlinear Functional Analysis**

**Nonlinear Functional Analysis and Applications**

New applications, research, and fundamental theories in nonlinear analysis are presented in this book. Each chapter provides a unique insight into a large domain of research focusing on functional equations, stability theory, approximation theory, inequalities, nonlinear
functional analysis, and calculus of variations with applications to optimization theory. Topics include: Fixed point theory Fixed-circle theory Coupled fixed points Nonlinear duality in Banach spaces Jensen's integral inequality and applications Nonlinear differential equations Nonlinear integro-differential equations Quasiconvexity, Stability of a Cauchy-Jensen additive mapping Generalizations of metric spaces Hilbert-type integral inequality, Solitons Quadratic functional equations in fuzzy Banach spaces Asymptotic orbits in Hill's problem Time-domain electromagnetics Inertial Mann algorithms Mathematical modelling Robotics Graduate students and researchers will find this book helpful in comprehending current applications and developments in mathematical analysis. Research scientists and engineers studying essential modern methods and techniques to solve a variety of problems will find this book a valuable source filled with examples that illustrate concepts.

**Geometric Nonlinear Functional Analysis**

This book presents a systematic and unified study of geometric nonlinear functional analysis. This is a very active research area and has connections to geometric measure theory, probability, classical analysis, combinatorics, and Banach space theory. Students and instructors alike benefit from examples and complete proofs.

**Topics in Nonlinear Functional Analysis**

A NATO Advanced Study Institute on Nonlinear Functional Analysis and Its Applications was held in Hotel Villa del Mare, Maratea, Italy during April 22 - May 3, 1985. This volume consists of the Proceedings of the Institute. These Proceedings include the invited lectures and contributed papers given during the Institute. The papers have been refereed. The aim of these lectures was to bring together recent and up-to-date development of the subject, and to give directions for future research. The main topics covered include: degree and generalized degree theory, results related to Hamiltonian Systems, Fixed Point theory, linear and nonlinear Differential and Partial Differential Equations, Theory of Nielsen Numbers, and applications to Dynamical Systems, Bifurcation Theory, Hamiltonian Systems, Minimax Theory, Heat Equations, Pendulum Equation, Nonlinear Boundary Value Problems, and Dirichlet and Neumann problems for elliptic equations and the periodic Dirichlet problem for semilinear beam equations. I express my sincere thanks to Professors F. E. Browder, R. Conti, A. Dowd, D. E. Edmunds and J. Mawhin members of the Advisory Committee.

**Nonlinear Functional Analysis and its Applications**

This second edition, like the first, attempts to arrive as simply as possible at some central problems in the Navier-Stokes equations in the following areas: existence, uniqueness, and regularity of solutions in space dimensions two and three; large time behavior of solutions and attractors; and numerical analysis of the Navier-Stokes equations. Since publication of the first edition of these lectures in 1983, there has been extensive research in the area of inertial manifolds for Navier-Stokes equations. These developments are addressed in a new section
devoted entirely to inertial manifolds. Inertial manifolds were first introduced under this name in 1985 and, since then, have been systematically studied for partial differential equations of the Navier-Stokes type. Inertial manifolds are a global version of central manifolds. When they exist they encompass the complete dynamics of a system, reducing the dynamics of an infinite system to that of a smooth, finite-dimensional one called the inertial system. Although the theory of inertial manifolds for Navier-Stokes equations is not complete at this time, there is already a very interesting and significant set of results which deserves to be known, in the hope that it will stimulate further research in this area. These results are reported in this edition.

**Nonlinear Differential Equations of Monotone Types in Banach Spaces**

Iterative Methods for Fixed Points of Nonlinear Operators offers an introduction into iterative methods of fixed points for nonexpansive mappings, pseudo-contractions in Hilbert Spaces and in Banach Spaces. Iterative methods of zeros for accretive mappings in Banach Spaces and monotone mappings in Hilbert Spaces are also discussed. It is an essential work for mathematicians and graduate students in nonlinear analysis.

**Nonlinear Functional Analysis and Its Applications**

This introduction to the ideas and methods of linear functional analysis shows how familiar and useful concepts from finite-dimensional linear algebra can be extended or generalized to infinite-dimensional spaces. Aimed at advanced undergraduates in mathematics and physics, the book assumes a standard background of linear algebra, real analysis (including the theory of metric spaces), and Lebesgue integration, although an introductory chapter summarizes the requisite material. A highlight of the second edition is a new chapter on the Hahn-Banach theorem and its applications to the theory of duality.

**Nonlinear Functional Analysis**

The contents of this monograph fall within the general area of nonlinear functional analysis and applications. We focus on an important topic within this area: geometric properties of Banach spaces and nonlinear iterations, a topic of intensive research efforts, especially within the past 30 years, or so. In this theory, some geometric properties of Banach spaces play a crucial role. In the first part of the monograph, we expose these geometric properties most of which are well known. As is well known, among all infinite dimensional Banach spaces, Hilbert spaces have the nicest geometric properties. The availability of the inner product, the fact that the proximity map or nearest point map of a real Hilbert space H onto a closed convex subset K of H is Lipschitzian with constant 1, and the following two identities $$2 \quad 2 \quad ||x+y|| = ||x|| + 2$$
x, y + ||y||, (??) 2 2 2 ||?x+(1??)y|| = ?||x|| + (1??)||y|| ??(1??)||x?y||, (??) which hold for all x, y ∈ H, are some of the geometric properties that characterize inner product spaces and also make certain problems posed in Hilbert spaces more manageable than those in general Banach spaces. However, as has been rightly observed by M. Hazewinkel, “many, and probably most, mathematical objects and models do not naturally live in Hilbert spaces”. Consequently, to extend some of the Hilbert space techniques to more general Banach spaces, analogues of the identities (??) and (??) have to be developed.

**Linear Functional Analysis**

This book consists of nine papers covering a number of basic ideas, concepts, and methods of nonlinear analysis, as well as some current research problems. Thus, the reader is introduced to the fascinating theory around Brouwer’s fixed point theorem, to Granas’ theory of topological transversality, and to some advanced techniques of critical point theory and fixed point theory. Other topics include discontinuous differential equations, new results of metric fixed point theory, robust tracker design problems for various classes of nonlinear systems, and periodic solutions in computer virus propagation models.

**Geometric Properties of Banach Spaces and Nonlinear Iterations**

Nonlinearity and Functional Analysis is a collection of lectures that aim to present a systematic description of fundamental nonlinear results and their applicability to a variety of concrete problems taken from various fields of mathematical analysis. For decades, great mathematical interest has focused on problems associated with linear operators and the extension of the well-known results of linear algebra to an infinite-dimensional context. This interest has been crowned with deep insights, and the substantial theory that has been developed has had a profound influence throughout the mathematical sciences. This volume comprises six chapters and begins by presenting some background material, such as differential-geometric sources, sources in mathematical physics, and sources from the calculus of variations, before delving into the subject of nonlinear operators. The following chapters then discuss local analysis of a single mapping and parameter dependent perturbation phenomena before going into analysis in the large. The final chapters conclude the collection with a discussion of global theories for general nonlinear operators and critical point theory for gradient mappings. This book will be of interest to practitioners in the fields of mathematics and physics, and to those with interest in conventional linear functional analysis and ordinary and partial differential equations.

**Banach Space Theory**

**Navier-Stokes Equations and Nonlinear Functional Analysis**
Nonlinear Functional Analysis and Its Applications

The aim of this book is to provide a concise but complete introduction to the main mathematical tools of nonlinear functional analysis, which are also used in the study of concrete problems in economics, engineering, and physics. This volume gathers the mathematical background needed in order to conduct research or to deal with theoretical problems and applications using the tools of nonlinear functional analysis.

Topological Methods in Nonlinear Functional Analysis

Banach spaces provide a framework for linear and nonlinear functional analysis, operator theory, abstract analysis, probability, optimization and other branches of mathematics. This book introduces the reader to linear functional analysis and to related parts of infinite-dimensional Banach space theory. Key Features: - Develops classical theory, including weak topologies, locally convex space, Schauder bases and compact operator theory - Covers Radon-Nikodým property, finite-dimensional spaces and local theory on tensor products - Contains sections on uniform homeomorphisms and non-linear theory, Rosenthal's L1 theorem, fixed points, and more - Includes information about further topics and directions of research and some open problems at the end of each chapter - Provides numerous exercises for practice. The text is suitable for graduate courses or for independent study. Prerequisites include basic courses in calculus and linear. Researchers in functional analysis will also benefit for this book as it can serve as a reference book.

Linear and Nonlinear Functional Analysis with Applications

This introduction to the ideas and methods of linear functional analysis shows how familiar and useful concepts from finite-dimensional linear algebra can be extended or generalized to infinite-dimensional spaces. Aimed at advanced undergraduates in mathematics and physics, the book assumes a standard background of linear algebra, real analysis (including the theory of metric spaces), and Lebesgue integration, although an introductory chapter summarizes the requisite material. The initial chapters develop the theory of infinite-dimensional normed spaces, in particular Hilbert spaces, after which the emphasis shifts to studying operators between such spaces. Functional analysis has applications to a vast range of areas of mathematics; the final chapters discuss the particularly important areas of integral and differential equations. Further highlights of the second edition include: a new chapter on the Hahn-Banach theorem and its applications to the theory of duality. This chapter also introduces the basic properties of projection operators on Banach spaces, and weak convergence of sequences in Banach spaces - topics that have applications to both linear and nonlinear functional analysis; extended coverage of the uniform boundedness theorem; plenty of exercises, with solutions provided at the back of the book.

Nonlinear Functional Analysis

Uncover the Useful Interactions of Fixed Point Theory with Topological Structures Nonlinear Functional Analysis in Banach Spaces and
Banach Algebras: Fixed Point Theory under Weak Topology for Nonlinear Operators and Block Operator Matrices with Applications is the first book to tackle the topological fixed point theory for block operator matrices with nonlinear entries in Banach spaces and Banach algebras. The book provides researchers and graduate students with a unified survey of the fundamental principles of fixed point theory in Banach spaces and algebras. The authors present several extensions of Schauder’s and Krasnosel'skii’s fixed point theorems to the class of weakly compact operators acting on Banach spaces and algebras, particularly on spaces satisfying the Dunford–Pettis property. They also address under which conditions a 2×2 block operator matrix with single- and multi-valued nonlinear entries will have a fixed point. In addition, the book describes applications of fixed point theory to a wide range of diverse equations, including transport equations arising in the kinetic theory of gas, stationary nonlinear biological models, two-dimensional boundary-value problems arising in growing cell populations, and functional systems of integral equations. The book focuses on fixed point results under the weak topology since these problems involve the loss of compactness of mappings and/or the missing geometric and topological structure of their underlying domain.

**Linear and Nonlinear Functional Analysis with Applications**

This monograph is concerned with the basic results on Cauchy problems associated with nonlinear monotone operators in Banach spaces with applications to partial differential equations of evolutive type. It focuses on major results in recent decades.

**Nonlinear Functional Analysis**

This single-volume textbook covers the fundamentals of linear and nonlinear functional analysis, illustrating most of the basic theorems with numerous applications to linear and nonlinear partial differential equations and to selected topics from numerical analysis and optimization theory. This book has pedagogical appeal because it features self-contained and complete proofs of most of the theorems, some of which are not always easy to locate in the literature or are difficult to reconstitute. It also offers 401 problems and 52 figures, plus historical notes and many original references that provide an idea of the genesis of the important results, and it covers most of the core topics from functional analysis.

**Applications of Nonlinear Analysis**

**Nonlinear Functional Analysis**

**Nonlinearity and Functional Analysis**
An Introduction to Nonlinear Functional Analysis and Elliptic Problems

Proceedings of the NATO Advanced Study Institute, Maratea, Italy, April 22-May 3, 1985

Linear Functional Analysis

This text offers a survey of the main ideas, concepts, and methods that constitute nonlinear functional analysis. It features extensive commentary, many examples, and interesting, challenging exercises. 1985 edition.

Topics in Nonlinear Functional Analysis

This book is based on the lectures presented at the Special Session on Nonlinear Functional Analysis of the American Mathematical Society Regional Meeting, held at New Jersey Institute of Technology. It explores global invertibility and finite solvability of nonlinear differential equations.

Nonlinear Functional Analysis and Differential Equations

Since its first appearance as a set of lecture notes published by the Courant Institute in 1974, this book has served as an introduction to various subjects in nonlinear functional analysis. The current edition is a reprint of these notes, with added bibliographic references. Topological and analytic methods are developed for treating nonlinear ordinary and partial differential equations. The first two chapters of the book introduce the notion of topological degree and develop its basic properties. These properties are used in later chapters in the discussion of bifurcation theory (the possible branching of solutions as parameters vary), including the proof of Rabinowitz's global bifurcation theorem. Stability of the branches is also studied. The book concludes with a presentation of some generalized implicit function theorems of Nash-Moser type with applications to Kolmogorov-Arnold-Moser theory and to conjugacy problems. After more than 20 years, this book continues to be an excellent graduate level textbook and a useful supplementary course text.

Journal of Nonlinear Functional Analysis and Differential Equations
This is an elementary and self-contained introduction to nonlinear functional analysis and its applications, especially in bifurcation theory.

**Spectral Theory and Nonlinear Functional Analysis**

This Research Note addresses several pivotal problems in spectral theory and nonlinear functional analysis in connection with the analysis of the structure of the set of zeroes of a general class of nonlinear operators. It features the construction of an optimal algebraic/analytic invariant for calculating the Leray-Schauder degree, new methods for solving nonlinear equations in Banach spaces, and general properties of components of solutions sets presented with minimal use of topological tools. The author also gives several applications of the abstract theory to reaction diffusion equations and systems. The results presented cover a thirty-year period and include recent, unpublished findings of the author and his coworkers. Appealing to a broad audience, Spectral Theory and Nonlinear Functional Analysis contains many important contributions to linear algebra, linear and nonlinear functional analysis, and topology and opens the door for further advances.

**Topics in Nonlinear Functional Analysis**

This volume contains the proceedings of the special session on Fixed Point Theory and Applications held during the Summer Meeting of the American Mathematical Society at the University of Toronto, August 21-26, 1982. The theory of contractors and contractor directions is developed and used to obtain the existence theory under rather weak conditions. Theorems on the existence of fixed points of nonexpansive mappings and the convergence of the sequence of iterates of nonexpansive and quasi-nonexpansive mappings are given. Degree of mapping and its generalizations are given in detail. A class of eventually condensing mappings is studied and multivalued condensing mappings with multiple fixed points are also given. Topological fixed points, including the study of the Nielsen number of a selfmap on a compact surface, extensions of a well-known result of Krasnoselskii's Compression of a Cone Theorem, are given. Also, fixed points, antipodal points, and coincidences of multifunctions are discussed. Several results with applications in the field of partial differential equations are given. Application of fixed point theory in the area of Approximation Theory is also illustrated.

**A Primer of Nonlinear Analysis**

This single-volume textbook covers the fundamentals of linear and nonlinear functional analysis, illustrating most of the basic theorems with numerous applications to linear and nonlinear partial differential equations and to selected topics from numerical analysis and optimization theory. This book has pedagogical appeal because it features self-contained and complete proofs of most of the theorems, some of which are not always easy to locate in the literature or are difficult to reconstitute. It also offers 401 problems and 52 figures, plus historical notes and many original references that provide an idea of the genesis of the important results, and it covers most of the core topics from functional analysis.
Nonlinear Functional Analysis and Its Applications

Some Topics in Nonlinear Functional Analysis

This introduction to the ideas and methods of linear functional analysis shows how familiar and useful concepts from finite-dimensional linear algebra can be extended or generalized to infinite-dimensional spaces. Aimed at advanced undergraduates in mathematics and physics, the book assumes a standard background of linear algebra, real analysis (including the theory of metric spaces), and Lebesgue integration, although an introductory chapter summarizes the requisite material. A highlight of the second edition is a new chapter on the Hahn-Banach theorem and its applications to the theory of duality.

Nonlinear Functional Analysis in Banach Spaces and Banach Algebras

This title presents background for the solution of non-linear equations in Banach spaces. It contains basic techniques in non-linear analysis and also touches upon today's research. The book deals with topics, such as measures on non-compactness, topological degree, and bifurcation theory.

Functional Analysis and Infinite-Dimensional Geometry

Nonlinear Functional Analysis and Applications provides information pertinent to the fundamental aspects of nonlinear functional analysis and its application. This book provides an introduction to the basic concepts and techniques of this field. Organized into nine chapters, this book begins with an overview of the possibilities for applying ideas from functional analysis to problems in analysis. This text then provides a systematic exposition of several aspects of differential calculus in norms and topological linear spaces. Other chapters consider the various settings in nonlinear functional analysis in which differentials play a significant role. This book discusses as well the generalized inverse for a bounded linear operator, whose range is not necessarily closed. The final chapter deals with the equations of hydrodynamics, which are usually highly nonlinear and difficult to solve. This book is a valuable resource for mathematicians. Readers who are interested in nonlinear functional analysis will also find this book useful.

Nonlinear Functional Analysis

Since its first appearance as a set of lecture notes published by the Courant Institute in 1974, this book served as an introduction to various subjects in nonlinear functional analysis. The current edition is a reprint of these notes, with added bibliographic references. Topological and analytic methods are developed for treating nonlinear ordinary and partial differential equations. The first two chapters of the book introduce
the notion of topological degree and develop its basic properties. These properties are used in later chapters in the discussion of bifurcation theory (the possible branching of solutions as parameters vary), including the proof of Rabinowitz global bifurcation theorem. Stability of the branches is also studied. The book concludes with a presentation of some generalized implicit function theorems of Nash-Moser type with applications to Kolmogorov-Arnold-Moser theory and to conjugacy problems. For more than 20 years, this book continues to be an excellent graduate level textbook and a useful supplementary course text. Titles in this series are copublished with the Courant Institute of Mathematical Sciences at New York University.